XLIII. *A Demonſtration of Two Theorems mentioned in Art.* XXV. *of the* Philoſophical Tranſactions *for the Year* 1775. *In a Letter from* Charles Hutton, *Eſq. F. R. S. to the Rev. Dr.* Horſley, *Sec. R. S.*

TO THE REV. DR. HORSLEY, SEC. R. S.

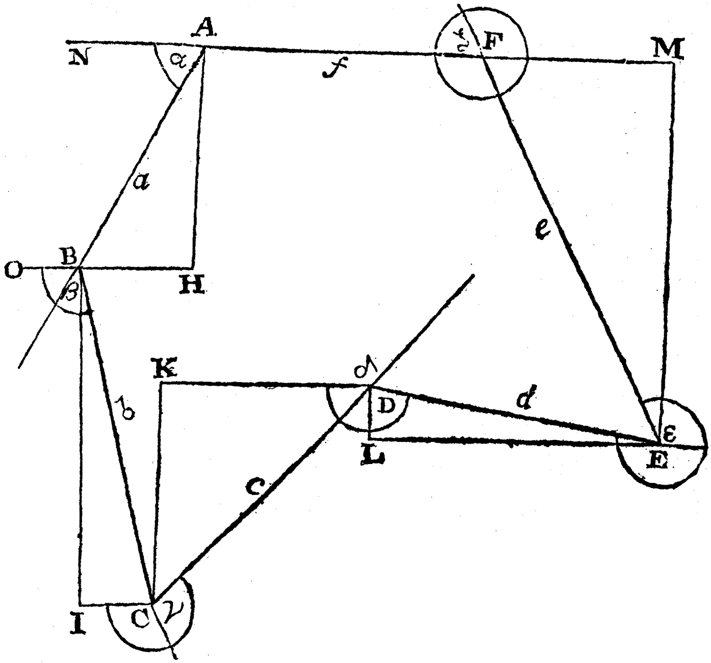
REV. SIR,

Woolwich Warren, Royal Mil. Acad.
April 17, 1776.

R. June 27, 1776.

THE following is a demonſtration of the two theorems mentioned in Art. XXV. of the Philoſophical Tranſactions for the year 1776. If you think them proper to be inſerted in the next volume of Tranſactions, be pleaſed to communicate them to the Royal Society for that purpoſe, from, &c.

THE

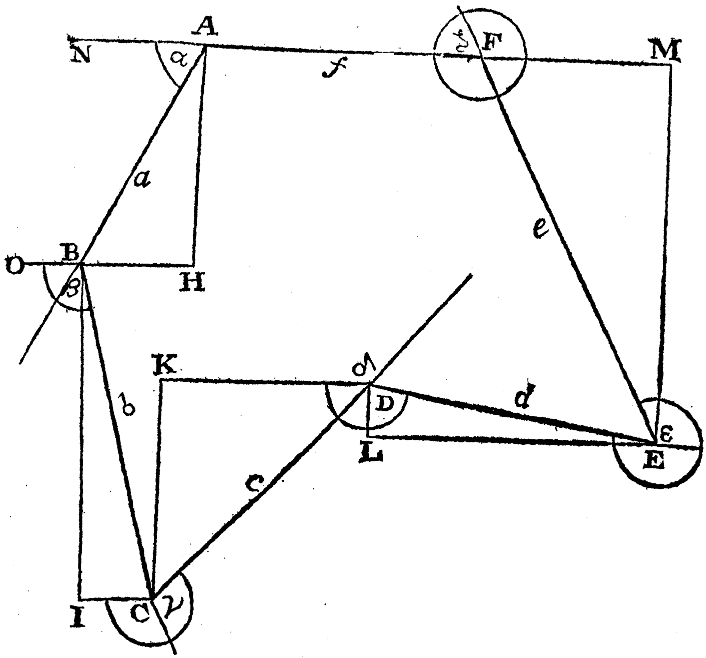


THE DEMONSTRATION.

Parallel to one ſide, AF, of the polygon ABCDEF, through all the angular points, draw BH, CI, DK, EL, FM. Then are the ſeveral angles, which are made by theſe parallels and the adjacent ſides of the figure, reſpectively equal to the ſums of all the exterior angles of the figure, from the beginning to the place of each angle; by the beginning is meant either extremity of the ſide AF. Thus the $\angle$ NAB $= \alpha$, the $\angle$ OBC $= \alpha + \beta$, the $\angle$ ICD $= \alpha + \beta + \gamma$, the $\angle$ KDE $= \alpha + \beta + \gamma - \delta$ (for here δ, being without the figure, is ſubtracted inſtead of being added), the $\angle$ LEF $= \alpha + \beta + \gamma - \delta + \varepsilon$, &c. All this is evident from Eucl. I. 29.; the angles we ſpeak of being thoſe, which are meaſured by the little arcs deſcribed about each angular point in the figure.

I. If right lines, AH, BI, CK, DL, EM, be drawn from the angular points, perpendicular each to the parallel which paſſes through the next angular point, the ſums of the perpendiculars, drawn in contrary directions, the one upwards, and the other downwards, will be equal. And each perpendicular will be a fourth in proportion with the radius, that ſide of the polygon which is adjacent both to the perpendicular and the parallel on which it falls, and the ſine of the ſum of the external angles taken to that incluſively from which the perpendicular is drawn. Thus, AH + BI + DL = CK + EM; and rad. ſin. α = AB : AH; and rad. ſin. $\overline{\alpha + \beta}$ = BC : BI, and in like manner of the reſt. Take the value, therefore, of each perpendicular by theſe analogies, putting unity for the radius, ſubtract the ſum of all that are drawn upwards from the ſum of all that are drawn downwards, and the remainder, put equal to o, is the firſt equation; that is, AH $= a \times$ ſ. α (for $\angle$ ABH $= \angle \alpha$); BI $= b \times$ ſ. $\overline{\alpha + \beta}$ (for ſ. $\angle$ ICB = ſ. $\angle$ HBC = ſ. of its ſuppl. OBC or $\alpha + \beta$); in like manner, CK $= - c \times$ ſ. $\overline{\alpha + \beta + \gamma}$; DL $= d \times$ ſ. $\overline{\alpha + \beta + \gamma - \delta}$; EM $= - e \times$ ſ. $\overline{\alpha + \beta + \gamma - \delta + \varepsilon}$, &c. the laſt perpendicular will always be = o, becauſe the ſine of 360°, or of $\overline{\alpha + \beta + \gamma - \delta + \varepsilon + \zeta}$ is nothing. Hence $a \times$ ſ. $\alpha + b \times$ ſ. $\overline{\alpha + \beta} + c \times$ ſ. $\overline{\alpha + \beta + \gamma} + d \times$ ſ. $\overline{\alpha + \beta + \gamma - \delta} + e \times$ ſ. $\overline{\alpha + \beta + \gamma - \delta + \varepsilon}$ = AH + BI − CK + DL − EM = o, which is the firſt equation.

II.



II. In like manner it appears, that the intercepted parts BH, CI, DK, EL, FM, AF, of the parallels before-mentioned, are equal to the feveral correfponding fides drawn into the cofines of the fame fums of the exterior angles (the radius being 1.); and becaufe BH − CI − DK − EL + FM + AF = 0, therefore $a \times$ cof. $\alpha + b \times$ cof. $\overline{\alpha+\beta} + c \times$ cof. $\overline{\alpha+\beta+\gamma} + d \times$ cof. $\overline{\alpha+\beta+\gamma-\delta} + e \times$ cof. $\overline{\alpha+\beta+\gamma-\delta+\varepsilon} + f \times$ cof. $\overline{\alpha+\beta+\gamma-\delta+\varepsilon+\zeta} = 0$, which is the fecond theorem. Or, for the laft term, $f \times$ cof. $\overline{\alpha+\beta+\gamma-\delta+\varepsilon+\zeta}$, of this latter theorem, might be fubftituted its value f only.